



## Math 118 - Fall 2022 - Common Final Exam, version A

Print name: \_\_\_\_\_

Section number: \_\_\_\_\_ Instructor's name: \_\_\_\_\_

**Directions:**

- This exam has 13 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.
- You may use a calculator which does not allow internet access. The use of any notes or electronic devices other than a calculator is prohibited.

**Good luck!**

Question:	1	2	3	4	5	6	7
Points:	12	9	3	6	11	6	8
Score:							
Question:	8	9	10	11	12	13	Total
Points:	9	9	6	9	6	6	100
Score:							

1. (12 points) The population of Aidan's rabbit farm is 1200 rabbits in the year 2022. He wants to know the future population so he can plan how many carrots to buy in the coming years.
- (a) If the population is growing at a rate of 20 rabbits per year, find a formula for the function  $P(t)$ , the number of rabbits  $t$  years after 2022. Your function should be in the form  $P(t) = mt + b$ .
- (b) If the population is increasing by 15% per year, find a formula for the function  $P(t)$ , the number of rabbits  $t$  years after 2022. Your function should be in the form  $P(t) = a \cdot b^t$ .
- (c) Using your work from part b, convert  $P(t)$  to the form  $P(t) = a \cdot e^{kt}$ . Round constants in your answer to three decimal places.
- (d) Using your work from either part b or part c, find the year that the population of the rabbit farm will hit 4800. Round to the nearest year.

2. (9 points) Melissa opens a bank account with an initial deposit of \$7000. It earns 5% nominal interest. Find the balance of her account after 3 years if interest is compounded as follows.
- (a) Annually (once a year).
  
  
  
  
  
  
  
  
  
  
  - (b) Weekly (52 times a year).
  
  
  
  
  
  
  
  
  
  
  - (c) Continuously.
3. (3 points) A bank offers a savings account that pays interest at a nominal rate of 9.4% per year. Find the effective annual rate if interest is compounded monthly (twelve times a year). That is, find the percentage that the amount in the bank account increases by each year. Round to three decimal places if necessary.
4. (6 points) The math department opens a pie stand outside of BVM Hall. Its profits are increasing by an annual growth factor of 1.19. Find the doubling time for the pie stand's profits. That is, find the amount of time it takes for the profits to double. Round to three decimal places if necessary.

5. (11 points) The number of farm cats that live on the Wilke family farm oscillates sinusoidally between a low of 4 on January 1st ( $t = 0$ ), and a high of 22 on July 1st ( $t = 6$ ).

(a) Find the amplitude, period, and midline of the function  $P = f(t)$ .

The amplitude is \_\_\_\_\_

The period is \_\_\_\_\_

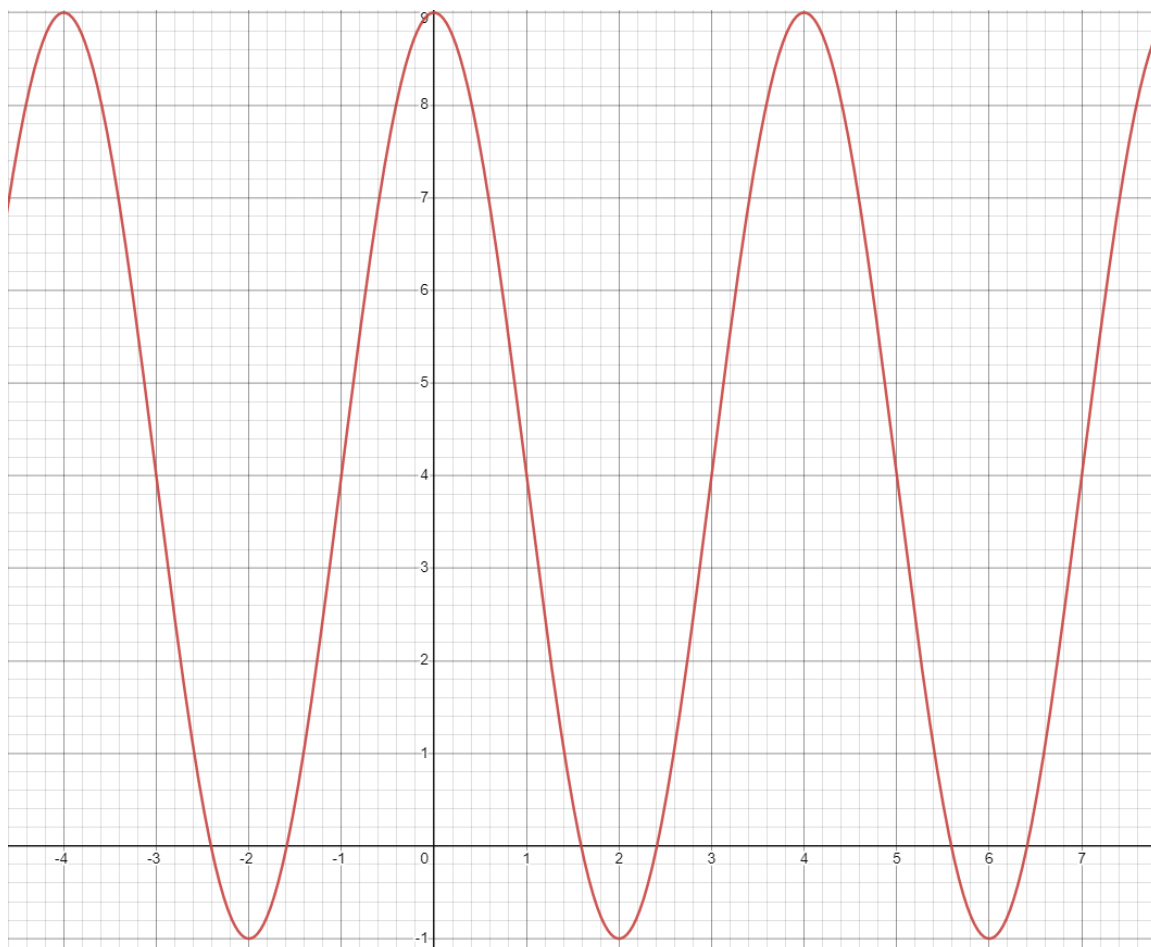
The midline is \_\_\_\_\_

(b) Find a formula for the population,  $P$ , in terms of time,  $t$ , in months since January 1st.

(c) Graph  $P$  as a function of  $t$ .



6. (6 points) Use the graph below of  $y = 5 \sin\left(\frac{\pi}{2}(t - 3)\right) + 4$  to approximate all solutions to the equation  $5 \sin\left(\frac{\pi}{2}(t - 3)\right) + 4 = 7$  on the interval  $[-2, 6]$ .



7. (8 points) For some  $\theta$ , such that  $\frac{3\pi}{2} \leq \theta \leq 2\pi$ , let  $\sin(\theta) = -\frac{4}{\sqrt{35}}$ .

(a) Find  $\cos(\theta)$ . Give an exact answer.

(b) Find  $\tan(\theta)$ . Give an exact answer.

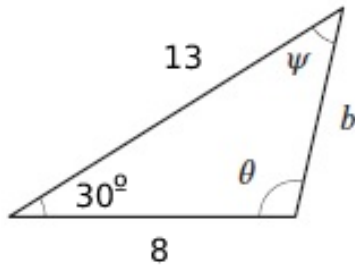
8. (9 points) A ladder is leaning against a building. The base of the ladder is 9 meters from the base of the building, and the ladder forms a  $63^\circ$  angle with the ground. The top of the ladder is exactly at the top of the building.

(a) Draw a picture of this situation.

(b) Find the height of the building. Round your answer to three decimal places.

(c) Find the length of the ladder. Round your answer to three decimal places.

9. (9 points) Find the missing sides and angle measure in the diagram below. Give angles as degrees, and round all answers to one decimal place.



10. (6 points) Perform the following conversions.

(a) Convert the Cartesian coordinates  $(4, 4)$  to polar coordinates. Give an exact answer.

(b) Convert the polar coordinates  $(3, \frac{\pi}{6})$  to Cartesian coordinates. Give an exact answer.

11. (9 points) Let  $P = f(t) = 800(1.062)^t$  be the number of students enrolled in Math 118, where  $t$  is measured in years since 2020.

(a) Evaluate  $f(20)$ . Round to the nearest whole number. Describe in words what this quantity tells you.

(b) Find a formula for  $f^{-1}(P)$  in terms of  $P$ . Give an exact answer.

(c) Evaluate  $f^{-1}(1600)$ . Round to the nearest whole number. Describe in words what this quantity tells you.

12. (6 points) Decompose the function

$$f(x) = \frac{15}{\sqrt{x-4}}$$

into a composition of two new functions  $u$  and  $v$ , where  $v$  is the inside function, that is  $f(x) = u(v(x))$ , so that  $u(x) \neq x$  and  $v(x) \neq x$ .

13. (6 points) What is the long-run behavior of the function given below?

(a) As  $x \rightarrow \infty, y = \frac{-2x(x-5)(x+3)}{8+x+x^2} \rightarrow$

(b) As  $x \rightarrow -\infty, y = \frac{-2x(x-5)(x+3)}{8+x+x^2} \rightarrow$